to be an aftringent mineral earth; whereas charcoal eafily takes fire, burns freely without fmoke, and continues burning, till it confumes to an afh; which confifts of an alkaline falt, and a pure earth, fit for making cuppels; and, by thefe marks, is fufficiently diftinguifhed from all mineral fubftances.

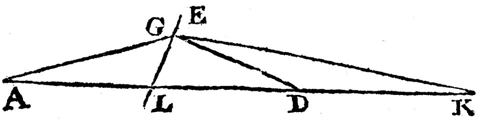
Grofvenor-Street, Jan. 8, 1761. Jer. Milles.

LXXXVI. *De Aberratione Luminis, in Superficiebus et Lentibus Sphæricis refractorum* *.

§ 1.

Read April 2, 1761.

SI radius luminis A G, incidens in fuperficiem quamcunque refringentem L E, inflectatur fecundum rectam G K, et quævis recta linea A K occurrat radio incidenti A G in A, refracto G K in K; et rectæ G D, normali ad fuperficiem refringentem L E, in D; erit rectangulum G K × D A ad rectangulum G A × D K, ut finus anguli incidentiæ D G A ad finum anguli refractionis D G K. Eft enim D A ad G A, ut fin. D G A ad fin. A D G, et G K : D K :: fin. G D K : fin. D G K; quare, ob



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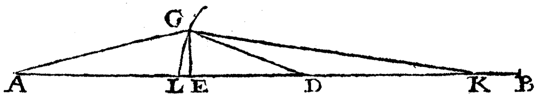
* This Paper, though fent to England in the fummer of the year 1760, was, by accident, prevented from being read to the Royal Society, till the 2d of April following.

fin.

sin. $ADG =$ sin. KDG, erit componendo $DA \times GK : GA \times DK ::$ sin. DGA : sin. DGK. Quæ ratio, si ponatur ut i ad r, erit $i.GA \times DK = r.GK \times DA$.

§ 2.

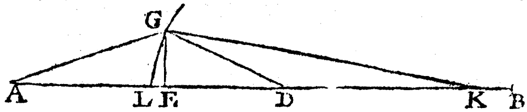
Incidat radius luminis AG in superficiem sphæricam LG, cujus centrum est D,



et refringatur secundum rectum GK, quæritur concursus K radii refracti GK cum axe sphæræ ALD, posito arcu LG satis parvo.

A puncto incidentiæ G cadat GE normalis ad axem AD, ducaturque radius sphæræ GD. His factis est per elementa, $AGq = ADq + DGq - 2AD \times DE = ADq + DLq - 2AD \times \overline{DL - LE} = \overline{AD - DL}q + 2AD \times LE = ALq + 2AD \times LE$, adeoque $AG = \sqrt{ALq + 2AD \times LE} =$ (ob LE satis parvam) $AL + \frac{AD \times LE}{AL}$ quamproxime. Similiter $KGq = KDq + DGq + 2KD \times DE = KDq + DLq + 2KD \times \overline{DL - LE} = \overline{KD + DL}q - 2DK \times LE = KLq - 2KD \times LE$, adeoque $KG = \sqrt{KLq - 2KD \times LE} = KL - \frac{KD \times LE}{KL}$ quamproxime.

Jam vero est (§ 1.) $i.GA \times DK = r.GK \times DA$, quare substitutis valoribus modo in-



ventis ipsarum GA et GK, habetur $i.KD \times (AL + \frac{AD \times LE}{AL}) = r.AD \times (KL - \frac{KD \times LE}{KL})$, et

trans-

transponendo $r.AD \times KL - i.KD \times AL = AD \times KD \times LE \times \left(\frac{i}{AL} + \frac{r}{KL}\right)$.

Ponatur jam $LA = A$, $LK = K$, radius sphæræ $DL = a$, adeoque $AD = A + a$, et $DK = K - a$. His in novissima æquatione scriptis, erit $rK \times \overline{A + a} - iA \times \overline{K - a} = \overline{A + a} \times \overline{K - a} \times LE \times \left(\frac{i}{A} + \frac{r}{K}\right)$, unde transponendo et dividendo, $K = \frac{iAa}{\overline{i - r}.A - ra} - \frac{\overline{A + a} \times \overline{K - a} \times LE}{\overline{i - r}.A - ra} \times \left(\frac{i}{A} + \frac{r}{K}\right)$.

Jam si radius incidens AG fuerit axi AD viciniſſimus, evanescet LE, adeoque et terminus $\frac{\overline{A + a} \times \overline{K - a} \times LE}{\overline{i - r}.A - ra} \times \left(\frac{i}{A} + \frac{r}{K}\right)$. Quare in hoc casu $K = \frac{iAa}{\overline{i - r}.A - ra}$, id est, distantia foci geometrici ipsi A conjugati à vertice L, erit $\frac{iAa}{\overline{i - r}.A - ra}$, unde aberratio radii refracti GK ab hoc foco erit $\frac{\overline{A + a} \times \overline{K - a} \times LE}{\overline{i - r}.A - ra} \times \left(\frac{i}{A} + \frac{r}{K}\right)$, sumendo à foco contra directionem cursus radiorum. Quoniam vero aberratio illa semper est valde parva, erit $\frac{iAa}{\overline{i - r}.A - ra}$ valor prope verus distantiæ LK sive K, adeoque in expressione aberrationis sine sensibili errore pro K usurpari potest. Sit itaque distantia LB foci geometrici B à vertice superficiei L, sive $\frac{iAa}{\overline{i - r}.A - ra} = B$,

eritque

eritque K ſive LB $= B - \frac{\overline{A+a}.\overline{B-a}.LE}{\overline{i-r}.A-ra} \times \left(\frac{i}{A}+\frac{r}{K}\right)$ quamproxime.

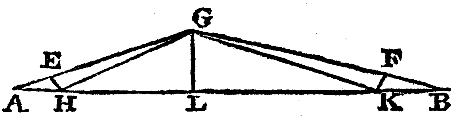
Dimidia latitudo ſuperficiei refringentis, ſive diſtantia puncti incidentiæ G ab axe, id eſt LG, dicatur L, erit quamproxime $LE = \frac{L^2}{2a}$: quare in formula modo inventa pro LE ſcribendo $\frac{L^2}{2a}$, et ulterius concinnando expreſſionem aberrationis ope æquationis $B = \frac{iAa}{\overline{i-r}.A-ra}$, habetur formula, qua in ſequentibus præcipue utemur: $LK = B - \frac{r.\overline{i-r}.B^2L^2}{2i^3} \times \left(\frac{1}{a}+\frac{1}{A}\right)^2 \times \left(\frac{r}{a}+\frac{i+r}{A}\right)$.

§ 3.

In formulis jam inventis ſigna ſymbolorum illi tantum caſui ſunt accommodata, quem figura exprimit, ubi radius luminis ab axe divergens ponitur incidere in ſuperficiem convexam, et poſt refractionem ad axem convergere. Accommodantur vero ad reliquos problematis caſus, mutando ſignum radii ſphæræ *a*, ſi radius luminis incidat in ſuperficiem concavam, et ſignum ipſius A, ſi radius incidens convergat ad axem. Hoc facto, ſi valor diſtantiæ LB ſive B prodit poſitivus, ſumenda eſt diſtantia illa à vertice L ſecundum directionem curſus radiorum; ſi negativus, contra eandem directionem. Aberratio vero radii à foco B, ſive BK per formulas computata, ſi fuerit poſitiva, ſumenda eſt à foco B contra directionem curſus radiorum; ſi negativa, ſecundum hanc directionem. Si radii incidentes fuerint axi paralleli, facienda eſt A infinita, et ſi B prodit infinita, radii refracti erunt axi paralleli, abſtrahendo ab illorum aberratione.

§ 4.

Si duo radii luminis AG, HG, in angulo quam minimo AGH inter ſe inclinati incidant fere perpendiculariter in idem punctum G ſuperficiei cujuſcunque refringentis LG, et refringantur, prior in GK, poſterior vero in GB, occurrentes lineæ rectæ cuicunque in punctis A, H, K, B; dico eſſe BK ad AH ut eſt $r.LBq$ ad $i.LAq$, poſita i ad r ut ſinus incidentiæ ad ſinum refractionis.

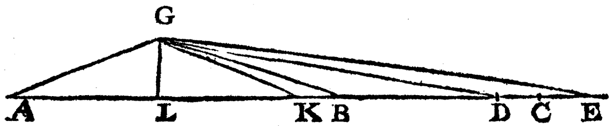


Quoniam enim anguli valde parvi ſunt ſinibus ſuis quamproxime proportionales, erunt in parvis refractionibus anguli incidentiæ et refractionis eorumque adeo differentiæ ſinibus incidentiæ et refractionis proportionales quamproxime. Eſt itaque angulus AGH, utpote differentia angulorum incidentiæ radiorum AG, HG, ad angulum BGK, ſive differentiam angulorum refractionis radiorum GK, GB, ut i ad r. Quare centro G inter hos angulos deſcriptis arcubus HE, KF, erit HD ad KF, ut $i.GH$ ad $r.GK$; ſed eſt $AH = \frac{GH.HE}{GL}$, et $BK = \frac{GK.KF}{GL}$, quare pro HE et KF, ſcribendo earum proportionales $i \times GH$ et $r \times GK$, habetur AH ad BK ut $i \times GHq$ ad $r \times GKq$, quamproxime, id eſt, ut $i \times LAq$ ad $r \times LBq$.

§ 5.

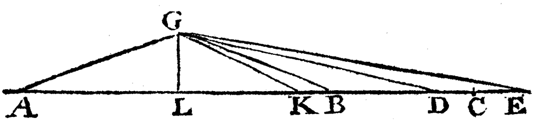
§ 5.

Data forma lentis refringentis, et data in ejus axe diſtantia foci radiorum incidentium, invenire punctum concurſus radii cujuſvis refracti cum axe.



Sit locus lentis in L, ejus axis ALBC, dimidia latitudo LG, ſive diſtantia incidentiæ ab axe, = L; radius convexitatis ſuperficiei primæ, in quam radii incidunt, = *a*; radius convexitatis ſuperficiei ſecundæ, è qua emergunt, = *b*, radius luminis quivis incidens AG, refractus GK. In axe lentis ABC ſit diſtantia foci radiorum incidentium, ſive LA = A, diſtantia foci geometrici ipſi A conjugati, ſive LB = B; diſtantia foci geometrici C ſuperficiei primæ ipſi A conjugati, ſive LC = C, et evidens eſt, eſſe etiam idem punctum C focum geometricum ſuperficiei ſecundæ conjugatum ipſi B. Sit tandem diſtantia puncti concurſus K radii refracti GK cum axe, ſive LK = K.

Radius AG in prima ſuperficie refractus tendat verſus punctum D, et concipiatur à puncto B, quod eſt focus lentis ipſi A conjugatis, radius BG incidere in ſecundam ſuperficiem, et in ea ita refringi, ut divergat à puncto axeos E. Et dabuntur per § 2. aberrationes CE et CD, adeoque earum ſumma DE. Sed per § 4. eſt DE ad BK ut $r \cdot LCq$ ad $i \cdot LBq$,



quare $BK = \frac{i.LBq}{r.LCq} \times DE = \frac{iB^2}{rC^2} \times DE$. Jam per § 2. est $CD = \frac{r.\overline{i-r}.C^2L^2}{2i^3} \times \left(\frac{1}{a} + \frac{1}{A}\right) \times \left(\frac{r}{a} + \frac{i+r}{A}\right)$ et $CE = \frac{r.\overline{i-r}.C^2L^2}{2i^3} \times \left(\frac{1}{b} + \frac{1}{B}\right) \times \left(\frac{r}{b} + \frac{i+r}{B}\right)$, quarum summa DE, ducta in $\frac{i.B^2}{r.C^2}$, dat aberrationem quæsitam $BK = \frac{iB^2}{rC^2} \times DE = \frac{\overline{i-r}.B^2L^2}{2i^2}\left[\left(\frac{1}{a} + \frac{1}{A}\right)^2 \times \left(\frac{r}{a} + \frac{\overline{i+r}}{A}\right) + \left(\frac{1}{b} + \frac{1}{B}\right)^2 \times \left(\frac{r}{b} + \frac{i+r}{B}\right)\right]$, quæ post evolutionem terminorum subtracta de LB, sive B, relinquit distantiam puncti concursus $LK =$

$$B - \frac{\overline{i-r}.B^2L^2}{2i^2} \times \left\{ \begin{array}{l} \frac{r}{a^3} + \frac{i+3r}{Aa^2} + \frac{2i+3r}{A^2a} + \frac{i+r}{A^3} \\ + \frac{r}{b^3} + \frac{i+3r}{Bb^2} + \frac{2i+3r}{B^2b} + \frac{i+r}{B^3} \end{array} \right\}.$$

Porro quoniam punctum C est focus ipsi A conjugatus respectu faciei primæ, erit $LC = \frac{iAa}{\overline{i-r}.A - ra}$; et quoniam idem punctum C est focus ipsi B conjugatus respectu superficiei secundæ, erit $LC = -\frac{iBb}{\overline{i-r}.B - rb}$, per § 2. Hinc $\frac{iAa}{\overline{i-r}.A - ra} = -\frac{iBb}{\overline{i-r}.B - rb}$, adeoque $B = \frac{rAab}{\overline{i-r}.A.\overline{a+b} - rab}$, sive $\frac{1}{A} + \frac{1}{B} = \frac{i-r}{r} \times \overline{\frac{1}{a} + \frac{1}{b}}$. Dato sic valore ipsius B, cognoscitur distantia concursus radii refracti GK cum axe sive LK, ejusque aberratio à foco B sive BK, quæ ob signum — sumenda est à foco B contra directionem

directionem cursus radiorum, si valorem habet positivum; secundum vero, si negativum.

Accommodantur vero hæ formulæ ad reliquos problematis casus eadem ratione ac dictum est § 3. de aberratione superficiei simplicis.

§ 6.

Expressio aberrationis jam inventa, ad formam reducitur usui magis accommodatam, conjungendo terminos, in quibus a et b easdem habent dimensiones, et reducendo singulas partes ope æquationis $\frac{i-r}{r} \times \overline{\frac{1}{a} + \frac{1}{b}} = \frac{1}{A} + \frac{1}{B} = \frac{1}{P}$, ubi P est distantia foci principalis. Hac ratione calculum subducendo habetur: $\frac{1}{a^3} + \frac{1}{b^3} = \frac{r}{\overline{i-r}\,.\,P} \times \left(\frac{r^2}{\overline{i-r}|^2\,.\,P^2} - \frac{3r}{\overline{i-r}\,.\,P\,.\,\overline{a+b}}\right)$; $\frac{1}{A a^2} + \frac{1}{B b^2} = \frac{r}{\overline{i-r}\,.\,P} \times \left(\frac{1}{Aa} + \frac{1}{Bb} - \frac{1}{P\,.\,\overline{a+b}}\right)$; $\frac{1}{A^2 a} + \frac{1}{B^2 b} = \frac{r}{\overline{i-r}\,.\,P} \times \left(\frac{i-r}{r} \times \overline{\frac{1}{Aa} + \frac{1}{Bb}} - \frac{1}{P\,.\,\overline{A+B}}\right)$; $\frac{1}{A^3} + \frac{1}{B^3} = \frac{r}{\overline{i-r}\,.\,P} \times \left(\frac{i-r}{rP^2} - \frac{3\,.\,\overline{i-r}}{rP\,.\,\overline{A+B}}\right)$.

Singulæ harum partium ductæ in suos respective coëfficientes, r, $i+3r$, $2i+r$, $i+r$, conjungantur cum signis propriis, et habebitur: $LK = LB - BK = B - \frac{B^2 L^2}{2 i P^2}\left(\frac{i^3 - 2i^2 r + 2r^3}{\overline{i-r}|^2\,.\,P} - \frac{3i+2r}{A+B} + \frac{2\,.\,\overline{i+r}}{A+B} \times \overline{\frac{A}{b} + \frac{B}{a}} - \frac{r\,.\,\overline{i+2r}}{\overline{i-r}\,.\,\overline{a+b}}\right)$, vel etiam $= B - \frac{B^2 L^2}{2 i P^2} \times \left(\frac{i^3}{\overline{i-r}|^2\,.\,P} - \frac{3i+2r}{A+B} - \frac{2\,.\,\overline{i+r}}{A+B} \times \overline{\frac{A}{a} + \frac{B}{b}} - \frac{r\,.\,\overline{i+2r}}{\overline{i-r}\,.\,\overline{a+b}}\right)$.

§ 7.

Invenire lentem, quæ radios luminis à data diſtantia advenientis in alia data diſtantia colligat cum minima aberratione, ipſamque aberrationem minimam.

Retentis omnibus ſymbolis ut ſupra, ſumantur differentialia aberrationis modo inventæ, $\frac{B^2L^2}{2i.P^2} \times \left(\frac{i^3}{\overline{i-r}|^2.P} - \frac{3i+2r}{A+B} - \frac{2.\overline{i+r}}{A+B} \times \overline{\frac{A}{a}+\frac{B}{b}} - \frac{r.\overline{i+2r}}{\overline{i-r}.\overline{a+b}}\right)$, poſitis a et b variabilibus, et habebitur $\frac{2.\overline{i+r}}{A+B} \times \overline{\frac{A\,da}{a^2} \times \frac{B\,db}{b^2}} + \frac{r.\overline{i+2r}}{i-r} \times \frac{da+db}{\overline{a+b}|^2} = 0$; ſed ob $\frac{1}{a}+\frac{1}{b} = \frac{r}{\overline{i-r}.P}$, eſt $\frac{da}{a^2}+\frac{db}{b^2} = 0$; quare tollendo per comparationem harum æquationum differentialia, invenitur: $\frac{b-a}{b+a} = \frac{2.\overline{i+r}.\overline{i-r}}{r.\overline{i+2r}} \times \frac{A-B}{A+B}$, adeoque $\frac{a}{b} = \frac{\overline{4r^2+ri-2i^2}.A+i.\overline{2i+r}.B}{\overline{4r^2+ri-2i^2}.B+i.\overline{2i+r}.A}$; unde per æquationem $\frac{1}{a}+\frac{1}{b} = \frac{r}{i-r} \times \overline{\frac{1}{A}+\frac{1}{B}}$, colliguntur radii facierum lentis quæſitæ, ſcil. $a = \frac{2.\overline{i-r}.\overline{i+2r}.AB}{i.\overline{2i+r}.A+\overline{4r^2+ri-2i^2}.B}$, et $b = \frac{2.\overline{i-r}.\overline{i+2r}.AB}{i.\overline{2i+r}.B+\overline{4r^2+ri-2i^2}.A}$; ſive $\frac{1}{a} = \frac{i.\overline{2i+r}}{2.\overline{i-r}.\overline{i+2r}} \times \frac{1}{B} + \frac{4r^2+ri-2i^2}{2.\overline{i-r}.\overline{i-2r}} \times \frac{1}{A}$; et $\frac{1}{b} = \frac{i.\overline{2i+r}}{2.\overline{i-r}.\overline{i+2r}} \times \frac{1}{A} + \frac{4r^2+ri-2i^2}{2.\overline{i-r}.\overline{i+2r}} \times \frac{1}{B}$.

Hic

Hic valores radiorum a et b ſubſtituti in formula aberrationis generali § 6. dant ipſam aberrationem minimam $= \frac{i \cdot B^2 L^2}{2 \cdot \overline{i + 2r} \cdot P^3} \times \left(\frac{P}{A + B} + \frac{r \cdot \overline{4i - r}}{4 \cdot \overline{i - r}^2} \right)$.

§ 8.

Poſtquam in § præcedente formam lentis invenimus, quæ in datis circumſtantiis aberrationem minimam producat, et ſimul ipſam ejus aberrationem; rationi conſentaneum eſt, inveſtigationem aberrationis lentium ab hac forma diſcendentium ita aggredi, ut ſimul appareat relatio formarum harum lentium ad illius formam, et aberrationis harum ad illius aberrationem. Solent enim concluſiones prodire concinniores, ſi ad caſum inter reliquos maxime ſingularem et unicum referantur, qualis hic eſt caſus aberrationis minimæ.

Scribatur brevitatis cauſa h pro $\frac{i \cdot \overline{2i + r}}{2 \cdot \overline{i - r} \cdot \overline{i + 2r}}$, et K pro $\frac{4r^2 + ri - 2i^2}{2 \cdot \overline{i - r} \cdot \overline{i + 2r}}$, et erit pro determinandis radiis a et b lentis aberrationis minimæ, $\frac{1}{a} = \frac{h}{B} + \frac{K}{A}$, et $\frac{1}{b} = \frac{h}{A} + \frac{K}{B}$, per § 7. Ponatur jam generaliter pro radiis a et b facierum cujuſvis alius lentis, $\frac{1}{a} = \frac{h}{B} + \frac{K}{A} + \frac{x}{P}$, et $\frac{1}{b} = \frac{h}{A} + \frac{K}{B} - \frac{x}{P}$; quos valores ipſarum $\frac{1}{a}$ et $\frac{1}{b}$ tales aſſumo, ut earum ſumma $\frac{1}{a} + \frac{1}{b}$ ſit $= \overline{h + K} \times \overline{\frac{1}{A} + \frac{1}{B}}$, ſive $\frac{r}{i - r} \times \overline{\frac{1}{A} + \frac{1}{B}}$, quemadmo-

dum

dum oportet. Subſtituantur hi valores ipſarum $\frac{1}{a}$ et $\frac{1}{b}$ in formula aberrationis generali, $\frac{B^2 L^2}{2 i P^2} \times \left(\frac{i^3}{P} - \frac{3i + 2r}{A + B}\right.$ $- \frac{2 . \overline{i + r}}{A + B} \times \overline{\frac{A}{a} + \frac{B}{b}} - \left.\frac{r . \overline{i + 2r}}{i - r . \overline{a + b}}\right)$ § 6. et peracto calculo invenietur $\frac{i B^2 L^2}{2 . \overline{i + 2r} . P^3} \left(\frac{P}{A + B} + \frac{\overline{4i - r} . r}{4 . \overline{i - r}^2}\right.$ $\left.\frac{\overline{i + 2r}^2}{i^2} x^2\right)$ pro aberratione lentis, cujus facies lumini obverſa radium habeat a, et facies à lumine averſa radium habeat b, exiſtentibus $\frac{1}{a} = \frac{h}{B} + \frac{K}{A} + \frac{x}{P}$, et $\frac{1}{b} = \frac{h}{A} + \frac{K}{B} - \frac{x}{P}$; P diſtantia foci principalis, A diſtantia foci radiorum incidentium, et B refractorum.

Commodum itaque hic accidit, ut numerus x, qui relationem formæ lentis ad lentem aberrationis minimæ deſignat, idem quoque relationem aberrationis ejus ad aberrationem minimam ratione non minus ſimplici exhibeat.

§ 9.

Si in formulis § præced. exhibitis, pro radiis facierum lentis ejuſque aberratione, ſtatuatur $x = 0$, radit lens ad eam formam, quæ aberrationem dat minimam, et aberratio ejus ad aberrationem minimam. Si vero non fuerit $x = 0$, apparet ex iiſdem formulis, cuivis aberrationis quantitati, quæ major ſit aberratione minima, duas diverſas reſpondere lentis formas, quarum utraque definitur æquationibus $\frac{1}{a} = \frac{h}{B} + \frac{K}{A} + \frac{x}{P}$, et $\frac{1}{a} = \frac{h}{A} + \frac{K}{B} - \frac{x}{P}$, ubi x pro

altera

altera positive, altera negative accipitur. Nam in formula aberrationis $\frac{iB^2L^2}{2.\overline{i+2r}.P^3} \times \left(\frac{P}{A+B} + \frac{\overline{4i-r}.r}{4.\overline{i-r}^2} + \frac{\overline{i+2r}^2}{i^2}x^2\right)$ non comparet nisi quadratum ipsius x, quod idem manet, sive radix x sumatur positive, sive negative.

Quoniam aberratio quævis $\frac{iB^2L^2}{2.\overline{i+2r}.P^3} \times \left(\frac{P}{A+B} + \frac{\overline{4i-r}.r}{4.\overline{i-r}^2} + \frac{\overline{i+2r}^2}{i^2}x^2\right)$ est ad aberrationem minimam $\frac{iB^2L^2}{2.\overline{i+2r}.P^3} \times \left(\frac{P}{A+B} + \frac{\overline{4i-r}.r}{4.\overline{i-r}^2}\right)$ in ratione majoris inæquabilitatis, statuatur illa ratio $1+m^2$ ad 1; assumendo m pro numero quovis, et invenitur $x = \pm \frac{mi}{i+2r}\sqrt{\left(\frac{P}{A+B} + \frac{\overline{4i-r}.r}{4.\overline{i-r}^2}\right)}$. Et sic ex data ratione aberrationis alicujus lentis ad aberrationem minimam, datur illi correspondens numerus x, adeoque et radii facierum ejus, per superiora.

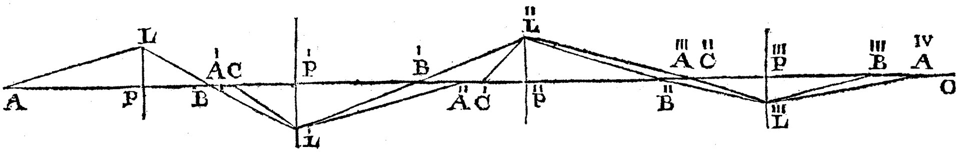
Numerus hic x, quoniam simul et formam et aberrationem suæ lentis indicat, dicatur *index* lentis, et scribatur brevitatis causa f pro $\frac{\overline{4i-r}.r}{4.\overline{i-r}^2}$, et g pro $\frac{1+2r}{i}$, ut aberratio sit $\frac{B^2L^2}{2g.P^3} \times \left(\frac{P}{A+B} + f + g^2x^2\right)$.

§ 10.

In linea recta A O, tanquam axe communi, dispositæ intelligantur lentes quotcunque P, $\overset{\text{I}}{\text{P}}$, $\overset{\text{II}}{\text{P}}$, $\overset{\text{III}}{\text{P}}$, &c. quarum foci conjugati sint respective, lentis

 primæ

primæ A et $\overset{I}{A}$; ſecundæ $\overset{I}{A}$ et $\overset{II}{A}$; tertiæ $\overset{II}{A}$ et $\overset{III}{A}$; quartæ $\overset{III}{A}$ et $\overset{IV}{A}$, &c.



Radius luminis, veniens à puncto A, refringatur in omnibus his lentibus in punctis L, $\overset{I}{L}$, $\overset{II}{L}$, $\overset{III}{L}$, &c. pergens ſecundum curſum A L $\overset{I}{L}$ $\overset{II}{L}$ $\overset{III}{L}$, et poſt ſingulas refractiones occurrat axi A O in punctis B, $\overset{I}{B}$, $\overset{II}{B}$, $\overset{III}{B}$, &c. aberrans à focis $\overset{I}{A}$, $\overset{II}{A}$, $\overset{III}{A}$, $\overset{IV}{A}$, &c. longitudinibus $\overset{I}{A}B$, $\overset{II}{A}\overset{I}{B}$, $\overset{III}{A}\overset{II}{B}$, $\overset{IV}{A}\overset{III}{B}$, &c. Quæruntur jam hæ aberrationes.

Lentium P, $\overset{I}{P}$, $\overset{II}{P}$, $\overset{III}{P}$, &c. diſtantiæ focorum principalium dicantur reſpective P, $\overset{I}{P}$, $\overset{II}{P}$, $\overset{III}{P}$, &c. et indices x, $\overset{I}{x}$, $\overset{II}{x}$, $\overset{III}{x}$, &c. Diſtantiæ focorum conjugatorum à ſuis reſpective lentibus ſint: pro lente prima P, $PA = A$, $P\overset{I}{A} = B$; pro lente ſecunda $\overset{I}{P}$, $\overset{I}{P}\overset{I}{A} = \overset{I}{A}$, $\overset{I}{P}\overset{II}{A} = \overset{I}{B}$; pro lente tertia $\overset{II}{P}$, $\overset{II}{P}\overset{II}{A} = \overset{II}{A}$, $\overset{II}{P}\overset{III}{A} = \overset{II}{B}$; pro lente quarta $\overset{III}{P}$, $\overset{III}{P}\overset{III}{A} = \overset{III}{A}$, et $\overset{III}{P}\overset{IV}{A} = \overset{III}{B}$, et ita porro: ponatur præterea P L, ſive diſtantia puncti incidentiæ in primam lentem ab axe $= L$, et erit $\overset{I}{P}\overset{I}{L} = \frac{\overset{I}{A}}{B} \times L$, $\overset{II}{P}\overset{II}{L} = \frac{\overset{I}{A} . \overset{II}{A}}{B . \overset{I}{B}} \times L$, $\overset{III}{P}\overset{III}{L} = \frac{\overset{I}{A} . \overset{II}{A} . \overset{III}{A}}{B . \overset{I}{B} . \overset{II}{B}} \times L$, et ita porro.

I. Vidimus in § 9. aberrationem $\overset{\text{I}}{A}B$ lentis primæ P esse $\frac{B^2 L^2}{2g . P^3} \times \left(\frac{P}{A+B} + f + g^2 x^2\right)$, ubi $f = \frac{\overline{4i - r} . r}{4 . \overline{i - r}|^2}$, et $g = \frac{i + 2r}{i}$.

II. Concipiatur à foco secundæ lentis $\overset{\text{II}}{A}$ ipsi $\overset{\text{I}}{A}$ conjugato in illam incidere radius $\overset{\text{II}}{A}\overset{\text{I}}{L}$, qui refractus secundum $\overset{\text{I}}{L}C$, conveniat cum axe in C, aberrans à foco $\overset{\text{I}}{A}$ longitudine $\overset{\text{I}}{A}C$: et erit per § 9. aberratio $\overset{\text{I}}{A}C = \frac{\overset{\text{I}}{A}^4 L^2}{2g . B^2 . P^3} \times \left(\frac{\overset{\text{I}}{P}}{\overset{\text{I}}{A} + \overset{\text{I}}{B}} + f + g^2 \overset{\text{I}}{x}^2\right)$, scribendo scilicet $\frac{\overset{\text{I}}{A}}{B} \times L$ pro L, $\overset{\text{I}}{B}$ pro A, $\overset{\text{I}}{A}$ pro B, et P pro P. Hæc aberratio $\overset{\text{I}}{A}C$, addita ad aberrationem primæ lentis modo inventam, $\overset{\text{I}}{A}B$, dat summam BC. Est vero BC ad $\overset{\text{I}}{B}\overset{\text{II}}{A}$ ut $\overset{\text{I}}{P}\overset{\text{I}}{A}q$ ad $\overset{\text{I}}{P}\overset{\text{II}}{A}q$, sive $\overset{\text{I}}{A}^2$ ad $\overset{\text{I}}{B}^2$, id est quantitates aberrationum à focis conjugatis sunt ut quadrata distantiarum focorum à lente, quod facile probari potest. Ergo aberratio $\overset{\text{II}}{A}\overset{\text{I}}{B}$, à refractione per duas lentes P et $\overset{\text{I}}{P}$ producta, habetur $= \frac{\overset{\text{I}}{B}^2}{\overset{\text{I}}{A}^2} \times BC$

$$= \frac{\overset{\text{I}}{B}^2 L^2}{2g} \times \left[\frac{B^2}{\overset{\text{I}}{A}^2 . P^3}\left(\frac{P}{A+B} + f + g^2 x^2\right) + \frac{\overset{\text{I}}{A}^4}{B^2 . \overset{\text{I}}{P}^3}\left(\frac{\overset{\text{I}}{P}}{\overset{\text{I}}{A} + \overset{\text{I}}{B}} + f + g^2 \overset{\text{I}}{x}^2\right)\right].$$

III. Concipiatur ſimiliter à foco interiori $\overset{\text{III}}{A}$ lentis tertiæ $\overset{\text{II}}{P}$ incidere in illam radium $\overset{\text{III}}{A}\,\overset{\text{II}}{L}$, cumque refractum convenire cum axe in puncto $\overset{\text{I}}{C}$, et aberrare à foco exteriore $\overset{\text{II}}{A}$ longitudine $\overset{\text{II}}{A}\,\overset{\text{I}}{C}$, quæ per § 9. eſt $\frac{\overset{\text{I}}{A}^2 . \overset{\text{II}}{A}^2 . L^2}{2g . B^2 . \overset{\text{I}}{B}^2 . \overset{\text{II}}{P}^3} \times \left(\frac{\overset{\text{II}}{P}}{\overset{\text{II}}{A} + \overset{\text{II}}{B}} + f + g^2 \overset{\text{II}}{x}^2\right)$. Hæc addita ad aberrationem duarum priorum lentium modo inventam, $\overset{\text{II}}{A}\,\overset{\text{I}}{B}$, dat ſummam $\overset{\text{II}}{A}\,\overset{\text{I}}{B} + \overset{\text{II}}{A}\,\overset{\text{I}}{C} = \overset{\text{I}}{B}\,\overset{\text{I}}{C}$, deinde fiat ut $\overset{\text{II}}{A}^2$ ad $\overset{\text{II}}{B}^2$; ita hæc ſumma $\overset{\text{I}}{B}\,\overset{\text{I}}{C}$ ad aberrationem trium lentium

$$\overset{\text{III}}{A}\,\overset{\text{II}}{B} = \frac{\overset{\text{II}}{B}^2 . L^2}{2g} \times \left[\frac{B^2 . \overset{\text{I}}{B}^2}{\overset{\text{II}}{A}^2 . \overset{\text{I}}{A}^2 . P^3} \left(\frac{P}{A+B} + f + g^2 x^2\right) + \frac{\overset{\text{I}}{B}^2\, \overset{\text{I}}{A}^2}{B^2 . \overset{\text{II}}{A}^2 . \overset{\text{I}}{P}^3} \left(\frac{\overset{\text{I}}{P}}{\overset{\text{I}}{A} + \overset{\text{I}}{B}} + f + g^2 \overset{\text{I}}{x}^2\right) + \frac{\overset{\text{I}}{A}^2 . \overset{\text{II}}{A}^2}{\overset{\text{I}}{B}^2 . B^2 . \overset{\text{II}}{P}^3} \left(\frac{\overset{\text{II}}{P}}{\overset{\text{II}}{A} + \overset{\text{II}}{B}} + f + g^2 \overset{\text{II}}{x}^2\right)\right].$$

IV. Eadem ratione invenitur aberratio $\overset{\text{IV}}{A}\,\overset{\text{III}}{B}$ poſt refractionem per quatuor lentes P, $\overset{\text{I}}{P}$, $\overset{\text{II}}{P}$, $\overset{\text{III}}{P}$, $=$

$$\frac{\overset{\text{III}}{B}^2 . L^2}{2g} \times \left[\frac{B^2 . \overset{\text{I}}{B}^2 . \overset{\text{II}}{B}^2}{\overset{\text{III}}{A}^2 . \overset{\text{II}}{A}^2 . \overset{\text{I}}{A}^2 . P^3} \times \left(\frac{P}{A+B} + f + g^2 x^2\right) + \frac{\overset{\text{I}}{B}^2 . \overset{\text{II}}{B}^2 . \overset{\text{I}}{A}^2}{B^2 . \overset{\text{III}}{A}^2 . \overset{\text{II}}{A}^2 . \overset{\text{I}}{P}^3} \times \left(\frac{\overset{\text{I}}{P}}{\overset{\text{I}}{A} + \overset{\text{I}}{B}} + f + g^2 \overset{\text{I}}{x}^2\right) + \frac{\overset{\text{II}}{B}^2 . \overset{\text{I}}{A}^2 . \overset{\text{II}}{A}^2}{\overset{\text{I}}{B}^2 . B^2 . \overset{\text{III}}{A}^2 . \overset{\text{II}}{P}^3} \times \left(\frac{\overset{\text{II}}{P}}{\overset{\text{II}}{A} + \overset{\text{II}}{B}} + f + g^2 \overset{\text{II}}{x}^2\right) + \frac{\overset{\text{I}}{A}^2 . \overset{\text{II}}{A}^2 . \overset{\text{III}}{A}^2}{\overset{\text{II}}{B}^2 . \overset{\text{I}}{B}^2 . B^2 . \overset{\text{III}}{P}^3} \left(\frac{\overset{\text{III}}{P}}{\overset{\text{III}}{A} + \overset{\text{III}}{B}} + f + g^2 \overset{\text{III}}{x}^2\right)\right].$$

Et eadem ratione progredi licet ad definiendam

definiendam aberrationem radiorum poſt tranſitum per plures lentes.

In his computationibus omnes lentes concipiuntur convexæ, et foci radiorum incidentium ponuntur jacere ante lentes ſuas, refractorum vero poſt lentes reſpectu curſus radiorum, quemadmodum figura exhibet. Formulæ autem allatæ ad quoſvis alios caſus facile accommodantur, mutando ſigna ſymbolorum, prout requirit mutatio hypotheſeos, quod monuiſſe ſufficiat.

§ 11.

Datis poſitionibus et focorum principalium diſtantiis lentium quotcunque, oporteat invenire formas earum, quæ efficiant, ut radii luminis, à dato in communi earum axe puncto advenientes, poſt refractionem colligantur in ultimo foco ſine abberratione, ſi fieri poteſt.

Solutio problematis generaliter abſolvitur, ponendo expreſſionem aberrationis, datæ multitudini lentium convenientem, (§ 10.) nihilo æqualem, quo ipſo habetur æquatio definiens rationem requiſitam indicum ſingularum lentium x, $\overset{\text{I}}{x}$, $\overset{\text{II}}{x}$, $\overset{\text{III}}{x}$, &c. Et quoniam relatio omnium indicum unica hac æquatione continetur, patet multum hic locum eſſe arbitrariæ nonnullorum aſſumtioni, prout requirunt vel ſuadent aperturæ lentium aliæque circumſtantiæ ex ſcopo inſtrumenti dijudicandæ, de quibus hic non eſt ſermo. In genere tantum cavendum eſt, ne aſſumtio ita fiat, ut aliquis indicum prodeat imaginarius. Determinatis ſic indicibus x, $\overset{\text{I}}{x}$, $\overset{\text{II}}{x}$, $\overset{\text{III}}{x}$, &c. inveniuntur radii facierum per § 8. Dicatur ſcilicet radius

faciei

faciei anterioris singularum lentium a, posterioris b; distantia foci principalis à lente, P; distantia foci radiorum incidentium ante lentem, A; distantia foci radiorum refractorum post lentem, B; et index ad lentem pertinens, x; et erit $\frac{1}{a} = \frac{h}{B} + \frac{K}{A} + \frac{x}{P}$, et $\frac{1}{b} = \frac{h}{A} + \frac{K}{B} - \frac{x}{P}$, ubi $h = \frac{i \cdot \overline{2i + r}}{2 \cdot \overline{i - r} \cdot \overline{i + 2r}}$, et $K = \frac{4r^2 + ri - 2i^2}{2 \cdot \overline{i - r} \cdot \overline{i + 2r}}$, unde dabuntur singularum lentium radii a et b.

Si nullus detur valor realis indicum x, $\overset{I}{x}$, $\overset{II}{x}$, $\overset{III}{x}$, &c. id quod accidit, si omnes æquationis termini ad unam partem rejecti fuerint ejusdem signi, casus propositus est impossibilis.

§ 12.

EXEMPLUM I. Ex duabus lentibus telescopium componere, quod sit liberum ab aberratione, et objecta visa amplificet in data ratione n ad 1. Abstrahitur autem hic a reliquis boni telescopii requisitis.

Lentes duæ primæ P et $\overset{I}{P}$ repræsentent binas lentes telescopii quæsiti, P quidem objectivam et $\overset{I}{P}$ ocularem (vid. fig. § 10.). Aberratio harum duarum lentium (§ 10. cas. 11.) nihilo æqualis posita dat æquationem generalem: $g^2 \times \left(\frac{B^2 \cdot x^2}{\overset{I}{A}{}^2 \cdot P^3} + \frac{\overset{I}{A}{}^2 \cdot \overset{I}{x}{}^2}{B^2 \cdot \overset{I}{P}{}^3}\right) + f \times \left(\frac{B^2}{\overset{I}{A}{}^2 \cdot P^3} + \frac{\overset{I}{A}{}^2}{B^2 \cdot \overset{I}{P}{}^3}\right) + \frac{B^2}{\overset{I}{A}{}^2 \cdot P^2 \cdot \overline{A + B}} + \frac{\overset{I}{A}{}^2}{B^2 \cdot \overset{I}{P}{}^2 \cdot \overline{\overset{I}{A} + \overset{I}{B}}} = 0$. Quoniam per constitutionem telescopii, radii luminis

minis in lentem objectivam incidentes, et è lente oculari emergentes, ſunt axi teleſcopii paralleli, erit A infinita, adeoque B = P; item $\overset{1}{B}$ infinita, adeoque $\overset{1}{A} = \overset{1}{P}$. Quare deletis duobus ultimis æquationis terminis, utpote evaneſcentibus, et in reliquis in ſcribendo P et $\overset{1}{P}$ pro B et $\overset{1}{A}$, habetur pro caſu hujus exempli æquatis $g^2 \times (P x^2 + \overset{1}{P} \overset{1}{x}{}^2) + f \times (P + \overset{1}{P}) = 0$, relationem indicum x et $\overset{1}{x}$ definiens, requiſitam ad id, ut aberratio radiorum è lente oculari emergentium nulla ſit. Jam ſi in hac æquatione P et $\overset{1}{P}$ ponantur ejuſdem ſigni, ſive utraque lens convexa, vel utraque concava, nullus valor realis haberi poteſt pro x et $\overset{1}{x}$, adeoque problema in hoc caſu erit impoſſibile. Statuatur itaque $\overset{1}{P}$ negativa, ſive lens ocularis concava, ſumendo pro exponente amplificationis objecti teleſcopio viſi, n, numerum quemvis poſitivum, et faciendo $\overset{1}{P} = -\frac{1}{n} P$, ut patet ex opticis. Scripto hoc valore pro $\overset{1}{P}$ in æquatione indicum, evadit illa $g^2 \times (\overset{1}{x}{}^2 - n x^2) = f \times \overline{n - 1}$. Horum itaque indicum utrumlibet pro lubitu aſſumere licet, modo ne $\overset{1}{x}$ fiat minor, quam $\frac{\sqrt{\overline{n - 1} . f}}{g}$, quod redderet indicem x imaginarium. Altero indicum ſic aſſumto, alter determinatur per allatam æquationem $g^2 \times (\overset{1}{x}{}^2 - n x^2) = f . \overline{n - 1}$. Quo facto radii facierum utriuſque lentis prodeunt per § 11. ut ſequitur:

Radius faciei anterioris lentis objectivæ $= \frac{P}{b + x}$, poſterioris $= \frac{P}{K - x}$;

Radius faciei anterioris lentis ocularis $= \frac{\overset{1}{P}}{K + \overset{1}{x}}$, poſterioris $= \frac{\overset{1}{P}}{b - \overset{1}{x}}$, ubi $\overset{1}{P}$ ſumenda eſt negativa, ſive $\overset{1}{P} = -\frac{1}{n} P$, ut dictum eſt.

Pro quovis itaque pari valorum correſpondentium indicum x et $\overset{1}{x}$, quatuor prodeunt diverſæ problematis ſolutiones, ob ſigna duplicia ipſarum x et $\overset{1}{x}$, unde duæ prodeunt formæ utriuſque lentis, quarum quælibet combinari poteſt cum binis formis alterius lentis.

Si detur vel aſſumatur forma alterutrius lentis, forma alterius per ſupradicta commode determinatur. Sit v. gr. lens objectiva plano-convexa, habens faciem convexam antrorſum verſam, planam retrorſum. Radius faciei ejus poſterioris $\frac{P}{K - x}$ ſtatuatur infinitus, et erit $x = K$, adeoque radius faciei, anterioris $\frac{P}{b + x} = \frac{P}{b + K} = \frac{i - r}{r} \times P$. Pro x ſcribatur K in æquatione indicum $\overset{1}{x}{}^2 - n x^2 = \frac{\overline{n - 1} . f}{g^2}$, et erit $\overset{1}{x} = \sqrt{\frac{\overline{n - 1} . f}{g^2} + n K^2}$, unde dantur radii facierum lentis ocularis, idque dupliciter, ob ſignum duplex valoris ipſius $\overset{1}{x}$.

Vidimus

Vidimus in solutione hujus problematis, aberrationem à duabus lentibus productam non posse evanescere, si utraque fuerit convexa, vel utraque concava, cujuscumque sint formæ et quomodocunque componantur. Idem quoque obtinet in quacunque lentium multitudine, quæ omnes sint convexæ, vel omnes concavæ. Omnes enim lentes, quarum facies superficierum sphæricarum sunt segmenta, aberrationes a focis suis eo producunt, quod radios luminis refractione nimium inflectunt; unde facile perspicitur, lentem convexam, convexis additam, vel concavam concavis, errores a prioribus productos augere. Quare ut aberratio a foco ultimo evanescat, debebunt lentium aliæ esse convexæ, aliæ concavæ, quo nimiæ radiorum incurvationes in unam partem corrigantur per nimias incurvationes factas in partem contrariam.

§ 13.

Exemplum II. Propositum sit investigare formas binarum lentium, quæ juxta se positæ radios incidentes axi parallelos in data à lentibus distantia R colligant, citra aberrationem ex figura facierum sphærica oriundam.

Quoniam in casu hujus exempli radii incidentes ponuntur paralleli erit in æquatione (§ 10. cas. 2.) pro relatione binorum indicum x et $\overset{1}{x}$, A infinita, adeoque B = P. Et quoniam lentes ponuntur juxta se positæ, erit earum distantia $B + \overset{1}{A} = 0$, sive $\overset{1}{A} = -B = -P$. Præterea est etiam $\frac{1}{\overset{1}{B}} = \frac{1}{P} + \frac{1}{\overset{1}{P}}$, unde $\overset{1}{B} - P = -\frac{P^2}{P + \overset{1}{P}}$. Substitutis itaque his valoribus, erit æquatio pro relatione indicum in caſu præſenti: $g^2 . (\overset{1}{P}^3 . x^2 + P^3 . \overset{1}{x}^2) + f . P^3 + \overset{1}{P}^3 = P . \overset{1}{P} . \overline{P + \overset{1}{P}}$, ſive $P^3 \overset{1}{x}^2 + \overset{1}{P}^3 x^2 = \frac{P + \overset{1}{P}}{g^2} \times (\overline{1+f} . P\overset{1}{P} - f . P^2 + \overset{1}{P}^2)$, quæ aſſumto numero quovis n, et ſtatuendo $\overset{1}{P} = nP$, evadit $\overset{1}{x}^2 + n^3 x^2 = \frac{n+1}{g^2} \times (n - \overline{n^2 - n + 1} . f)$. Ne autem caſus propoſitus fiat impoſſibilis, ob utramque lentem convexam, vel utramque concavam, per n ſemper intelligi debet numerus negativus. Aſſumatur jam pro Cubitu alteruter indicum x et $\overset{1}{x}$, et dabitur alter per æquationem allatam.

Determinato vero utroque indice, radii facierum erunt ut ſequitur:

Radius faciei anterioris lentis primæ $= \frac{P}{h + x}$, poſterioris $= \frac{P}{K - x}$;

Radius faciei anterioris lentis ſecundæ $= \frac{\overset{1}{P}}{\overline{n+1} . h - nK + \overset{1}{x}}$, poſterioris $= \frac{\overset{1}{P}}{\overline{n+1} . K - nh - \overset{1}{x}}$, ubi eſt $nP = \overset{1}{P}$, poſito n numero negativo.

Diſtantia foci principalis ſyſtematis binarum lentium P et $\overset{1}{P}$ dicatur R, et quoniam eſt $\frac{1}{R} = \frac{1}{P} + \frac{1}{\overset{1}{P}}$, erit $P = \frac{n+1}{n} R$, et $\overset{1}{P} = \overline{n+1} . R$, quare in expreſſionibus radiorum ſcribendo hos valores pro P et $\overset{1}{P}$, habebuntur:

Radius faciei anterioris lentis primæ $= \frac{\frac{n+1}{n}R}{h+x}$,

posterioris $= \frac{\frac{n+1}{n}R}{K-x}$;

Radius faciei anterioris lentis secundæ $=$ $\frac{\overline{n+1}.R}{\overline{n+1}.h-nK+\overset{1}{x}}$, posterioris $= \frac{\overline{n+1}.R}{\overline{n+1}.K-nh-\overset{1}{x}}$.

Et hæ lentes juxta se positæ radios incidentes axi parallelos in distantia à lente, R, colligent sine aberratione, si distantia foci lentis concavæ major fuerit distantia foci lentis convexæ; si autem contra dispergentur radii sine aberratione à puncto ante lentem, cujus distantia positive sumta est R.

Pro casu singulari hujus exempli, ponamus requiri formas binarum lentium ita comparatarum, ut distantia foci principalis lentis concavæ sit ad distantiam foci principalis lentis convexæ, ut 3 ad 2, et ut radii colligantur in distantia R à lentibus. Si lens anterior debet esse convexa, erit $n = -\frac{3}{2}$, et æquatio definiens relationem indicum lentis convexæ x et lentis concavæ $\overset{1}{x}$, $8\overset{1}{x}^2 - 27x^2 = \frac{19f+6}{g^2}$. Determinatis vero decenter indicibus x et $\overset{1}{x}$, radii facierum erunt, ut sequitur:

Radius faciei anterioris lentis primæ $= \frac{R}{3h+3x}$,

posterioris $= \frac{R}{3K-3x}$;

Radius faciei anterioris lentis secundæ $=$ $-\frac{R}{3K-h+2\overset{1}{x}}$, posterioris $= -\frac{R}{3h-K-2\overset{1}{x}}$.

 Si

Si vero lens anterior debet esse concava, erit $n = -\frac{2}{3}$, et relatio indicum $8x^2 - 27\overset{\text{I}}{x}{}^2 = \frac{19f + 6}{g^2}$, radii vero facierum ut sequitur:

Radius faciei anterioris lentis primæ $= -\frac{R}{2h + 2x}$, posterioris $= -\frac{R}{2K - 2x}$;

Radius faciei anterioris lentis secundæ $= \frac{R}{h + 2K + 3\overset{\text{I}}{x}}$, posterioris $= \frac{R}{K + 2h - 3\overset{\text{I}}{x}}$.

§ 14.

Exemplum III. Propositum sit, telescopium ex tribus lentibus componere, quarum prima P et secunda $\overset{\text{I}}{P}$ juxta se positæ vitrum objectivum duplex constituant, cujus distantia foci principalis sit R; tertia vero sive ocularis $\overset{\text{II}}{P}$, habeat distantiam foci $\overset{\text{II}}{P} = \frac{R}{m}$, ut potentia amplificandi designetur per numerum m. Ponatur præterea $\overset{\text{I}}{P} = nP$, intelligendo per n numerum datum negativum, eumque unitate majorem, si lens convexa anterior est collocanda, minorem si posterior, ut radii post refractionem in vitro objectivo convergant, quemadmodum expositum est in exemplo præcedente. Quæruntur autem formæ harum trium lentium, ut telescopium liberum sit ab omni erratione ex figura illarum sphærica oriunda.

Aberratio à tribus lentibus P, $\overset{\text{I}}{P}$, $\overset{\text{II}}{P}$, oriunda (§ 10. cas. III.) nihilo æqualis posita, dat $\frac{B^2 . \overset{\text{I}}{B}{}^2}{\overset{\text{II}}{A}{}^2 . \overset{\text{I}}{A}{}^2 . P^3} \left(\frac{P}{A + B}\right.$

+

$$+f+g^2 x^2) + \frac{\overset{I}{B}{}^2 . \overset{I}{A}{}^2}{B^2 . \overset{II}{A}{}^2 . \overset{I}{P}{}^3}\left(\frac{\overset{I}{P}}{\overset{I}{A}+\overset{I}{B}}+f+g^2\overset{I}{x}{}^2\right)$$

$$+\frac{\overset{I}{A}{}^2 . \overset{II}{A}{}^2}{\overset{I}{B}{}^2 . B^2 . \overset{II}{P}}\left(\frac{\overset{II}{P}}{\overset{II}{A}+\overset{II}{B}}+f+g^2\overset{II}{x}{}^2\right)=0.$$

Symbola hanc æquationem ingredientia per hypothesin hujus exempli ita determinantur: ob A infinitam est $\frac{P}{A+B}=0$, et $B=P$. Ob $\overset{II}{B}$ infinitam est $\frac{\overset{II}{P}}{\overset{II}{A}+\overset{II}{B}}=0$, et $\overset{II}{A}=\overset{II}{P}$. Ob lentes P et $\overset{I}{P}$ juxta se positas; est earum distantia $\overset{I}{A}+B=0$, sive $\overset{I}{A}=-B=-P$. Ob $\frac{1}{R}=\frac{1}{P}+\frac{1}{\overset{I}{P}}$, et $\overset{I}{P}=nP$, est $P=\frac{n+1}{n}R$, et $\overset{I}{P}=\overline{n+1}.R$. Præterea est $\overset{I}{B}=R$, et $\overset{II}{P}=\frac{R}{m}$. His itaque valoribus substitutis, et æquatione ordinata, habetur relatio indicum x, $\overset{I}{x}$, $\overset{II}{x}$, expressa ut sequitur: $g^2 \times (n^3x^2+\overset{I}{x}{}^2+\frac{\overline{n+1}^3}{m}\overset{II}{x}{}^2)$ $+f\times(n^3+1+\frac{\overline{n+1}^3}{m})=n.\overline{n+1}$. Determinatis indicibus x, $\overset{I}{x}$, $\overset{II}{x}$, habentur per § 11. radii facierum: videlicet,

Radius faciei anterioris lentis primæ $=\frac{\overline{n+1}.R}{n.\overline{h+x}}$, posterioris $=\frac{\overline{n+1}.R}{n.\overline{K-x}}$;

Radius faciei anterioris lentis secundæ $=\frac{\overline{n+1}.R}{\overline{n+1}.h-nK+\overset{I}{x}}$, posterioris $=\frac{\overline{n+1}.R}{\overline{n+1}.K-nh-\overset{I}{x}}$;

Radius

Radius faciei anterioris lentis tertiæ $= \frac{R}{m \cdot K + \overset{II}{x}}$, posterioris $= \frac{R}{m \cdot h - \overset{II}{x}}$.

Ita si vitrum objectivum hujus telescopii componendum sit ex duabus lentibus, convexa et concava, quarum distantiæ focales sint ut 3 et 2, debeatque primo lens convexa anterior collocari, oportebit sumere $n = -\frac{3}{2}$, et erit æquatio pro indicibus:

$$g^2 \times \left(27 x^2 - 8 \overset{I}{x}{}^2 + \frac{\overset{II}{x}{}^2}{m}\right) + f \times \left(19 + \frac{1}{m}\right) + 6 = 0,$$ et radii facierum:

Radius faciei anterioris lentis primæ $= \frac{R}{3h + 3x}$, posterioris $= \frac{R}{3K + 3x}$;

Radius faciei anterioris lentis secundæ $= -\frac{R}{3K - h + 2\overset{I}{x}}$, posterioris $= -\frac{R}{3h - K - 2\overset{I}{x}}$;

Radius faciei anterioris lentis tertiæ $= \frac{R}{mK + m\overset{II}{x}}$, posterioris $= \frac{R}{mh - m\overset{II}{x}}$.

Si vero lens concava anterior est collocanda, debet sumi $n = -\frac{2}{3}$, unde æquatio pro indicibus prodit

$$g^2 \times \left(-8 x^2 + 27 \overset{I}{x}{}^2 + \frac{\overset{II}{x}{}^2}{m}\right) + f \times \left(19 + \frac{1}{m}\right) + 6 = 0,$$ et consequenter:

Radius faciei anterioris lentis primæ $= -\frac{R}{2h + 2x}$, posterioris $= -\frac{R}{2K - 2x}$;

Radius

Radius faciei anterioris lentis secundæ $= \frac{R}{h + 2K + 3\overset{I}{x}}$, posterioris $= \frac{R}{K + 2h - 3\overset{I}{x}}$;

Radius faciei anterioris lentis tertiæ $= \frac{R}{mK + m\overset{II}{x}}$,

posterioris $= \frac{R}{mh - m\overset{II}{x}}$.

§ 15.

Vidimus in solutione generati novissimi problematis (§ ii.) et subjunctis exemplis (§ 12, 13, 14.) relationem indicum x, $\overset{I}{x}$, $\overset{II}{x}$, &c. unica tantum æquatione definiri, adeoque judicio artificis plurimum esse relictum in commoda eorum assumtione facienda. Duæ autem sunt considerationes, quibus hæc assumtio utcunque dirigi possit. Prima est, ut nullus indicum fiat nimis magnus, sive, quod eodem recidit, ut formæ lentium quamproxime accedant ad eas, quæ in datis circumstantiis aberrationem dant omnium minimam. Hac enim ratione errores in forma lentium forte commissi minus nocebunt, quemadmodum ex natura minimi notum est. Altera est, ut singulæ lentes, quam fieri potest, proxime evadant æqualiter utrinque convexæ, vel æqualiter concavæ, quo fiet, ut majores aperturæ sint patientes. Quantitates enim aberrationum supra definitæ non sunt nisi quamproxime veræ, et tanto magis fallunt, quanto facies lentium fuerint majora suarum sphærarum segmenta. Binæ hæ regulæ, etiamsi sibi invicem sæpius adversuntur, nec facile definiri queat, quantum in quovis dato casu uni vel alteri ipsarum sit tribuendum; non dubito tamen, quin judicioso artifici sint profuturæ

profuturæ ad mediam viam tenendam inter utramque.

Eædem quoque considerationes impediunt, quo minus certi aliquid definire ausim, de maximo possibili effectu instrumentorum opticorum secundum hæc principia compositorum, vel de modo perfectissima componendi. Regulæ huc pertinentes tutissime exspectantur ab experientia, modo dirigatur à theoria.

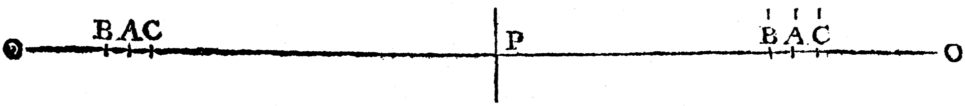
§ 16.

Expositis jam breviter quæ dicenda habuimus de aberrationibus radiorum homogeneorum refractorum, ortis a lentium figura sphærica, ut et de modo illas corrigendi in instrumentis opticis: liceat etiam pauca addere de altera illa aberrationis specie, quæ a diversa radiorum refrangibilitate oritur, quæ objecta trans lentes visa coloribus inficit, et maximam confusionis partem producit. Creditum hucusque communiter fuit hoc vitium nulla arte emendabile, donec ingeniosissimus artifex Londinensis, Dollandus, experimentis institutis feliciter deprehenderet, varia dari vitrorum genera, quæ, licet fere æqualiter refringant certum quoddam radiorum genus, plurimum tamen discrepent viribus refringendi reliqua radiorum genera, adeoque et radios heterogeneos a se invicem separandi: quo eximio invento et theoriam et praxim optices insigniter auxit. Discimus enim hinc, rationes refractionem radiorum diversi generis nullo modo a se invicem dependere, ut hucusque fuit creditum; adeoque frustra quæri regulam, qua ex datis refractionibus radiorum diversi generis in uno aliquo medio inveniantur refractiones eorumdem in alio;
vel

vel qua ex data refractione unius alicujus radii in quovis medio, inveniuntur refractiones reliquorum in eodem medio. Praxis quoque telescopiorum dioptricorum insigne incrementum hujus inventi beneficio nacta est. Scilicet sagacissimus inventor modum inde derivavit corrigendi aberrationes ex separatione radiorum heterogeneorum oriundas, componendo vitrum objectivum ex duabus lentibus, una convexa, altera concava, quæ ita sint comparatæ, ut una alterius effectum in radiis separandis destruat, quemadmodum in Vol. L. Part. II. p. 733. et seq. Transactionum Philosophicarum exposuit. Quod artificium ita breviter explicabimus.

§ 17.

Sit P lens quæcunque, habens radios facierum suarum *a* et *b*; distantiam foci principalis mediocriter refrangibilium P; et *a x* in rectam O O.



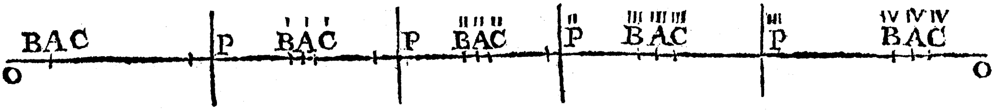
Ratio refractionis radiorum mediocriter refrangibilium ex aëre in lentem sit N ad 1; radiorum maxime refrangibilium N + *n* ad 1, et radiorum minime refrangibilium N — *n* ad 1, existente *n* numero valde parvo respectu ipsius N. In axe lentis O O sint puncta A, B, C foci radiorum in lentem P incidentium, A mediocriter refrangibilium, B maxime et C minime refrangibilium, quorum focorum distantiæ A B, A C sint valde parvæ, et ponantur puncta $\overset{1}{A}$, $\overset{1}{B}$, $\overset{1}{C}$, foci conjugati ipsorum A, B, C respective, pro sua quique radiorum specie, ita ut sint B C, $\overset{1}{B}\overset{1}{C}$,

 dissipa-

dissipationes radiorum heterogeneorum circa focos conjugatos A et $\overset{1}{A}$. Quæratur earum relatio.

Quoniam B et $\overset{1}{B}$ sunt foci conjugati radiorum maxime refrangibilium, quorum ratio refractionis est N + n ad 1, erit $\frac{1}{PB} + \frac{1}{P\overset{1}{B}} = \overline{N+n-1} \cdot \overline{\frac{1}{a} + \frac{1}{b}}$. Similiter quoniam puncta C et $\overset{1}{C}$ sunt foci conjugati radiorum minime refrangibilium, quorum ratio refractionis est N — n ad 1, erit $\frac{1}{PC} + \frac{1}{P\overset{1}{C}} = \overline{N-n+1} \cdot \overline{\frac{1}{a} + \frac{1}{b}}$. Subtrahatur hæc posterior æquatio à priore, et habebitur $\frac{1}{PB} - \frac{1}{PC} + \frac{1}{P\overset{1}{B}} - \frac{1}{P\overset{1}{C}} = 2n \cdot \overline{\frac{1}{a} + \frac{1}{b}}$, sive conjungendo binos terminos: $\frac{PC - PB}{PB \times PC} + \frac{P\overset{1}{C} - P\overset{1}{B}}{P\overset{1}{B} \times P\overset{1}{C}} = 2n \cdot \overline{\frac{1}{a} + \frac{1}{b}}$, quæ pro $PB \times PC$ et $P\overset{1}{B} \times P\overset{1}{C}$, scribendo PAq et $P\overset{1}{A}q$, et pro $\frac{1}{a} + \frac{1}{b}$ scribendo $\frac{1}{\overline{N-1} \cdot P}$, evadit $\frac{\overset{1}{B}\overset{1}{C}}{P\overset{1}{A}q} = \frac{2n}{\overline{N-1} \cdot P} + \frac{BC}{PAq}$, exhibens relationem inter dissipationes conjugatas BC et $\overset{1}{B}\overset{1}{C}$.

His præmissis, intelligantur lentes quotcunque P, $\overset{1}{P}$, $\overset{11}{P}$, $\overset{111}{P}$, &c. quarum distantiæ focorum principalium respectu radiorum mediocriter refrangibilium sint P, $\overset{1}{P}$, $\overset{11}{P}$, $\overset{111}{P}$, &c. respective, ordine dispositæ in axe earum



earum communi O O. Rationes refractionum radiorum mediocriter refrangibilium in his lentibus ordine sumtis sint respective, N ad 1, $\overset{I}{N}$ ad 1, $\overset{II}{N}$ ad 1, $\overset{III}{N}$ ad 1, &c. radiorum maxime refrangibilium $N + n$ ad 1, $\overset{I}{N} + \overset{I}{n}$ ad 1, $\overset{II}{N} + \overset{II}{n}$ ad 1, $\overset{III}{N} + \overset{III}{n}$ ad 1, &c. et radiorum minime refrangibilium $N - n$ ad 1, $\overset{I}{N} - \overset{I}{n}$ ad 1, $\overset{II}{N} - \overset{II}{n}$ ad 1, $\overset{III}{N} - \overset{III}{n}$ ad 1, &c. Puncta A, $\overset{I}{A}$, $\overset{II}{A}$, $\overset{III}{A}$, $\overset{IV}{A}$, &c. sint foci conjugati lentium respectu radiorum mediocriter refrangibilium, puncta B, $\overset{I}{B}$, $\overset{II}{B}$, $\overset{III}{B}$, $\overset{IV}{B}$, &c. respectu radiorum maxime refrangibilium, C, $\overset{I}{C}$, $\overset{II}{C}$, $\overset{III}{C}$, $\overset{IV}{C}$, &c. respectu minime refrangibilium, ita ut BC, $\overset{I}{B}\overset{I}{C}$, $\overset{II}{B}\overset{II}{C}$, $\overset{III}{B}\overset{III}{C}$, $\overset{IV}{B}\overset{IV}{C}$, &c. sint dissipationes successivæ radiorum heterogeneorum in axe lentium. Dicantur tandem distantiæ focorum conjugatorum à suis respective lentibus: $PA = A$, $P\overset{I}{A} = B$; $\overset{I}{P}\overset{I}{A} = \overset{I}{A}$, $\overset{I}{P}\overset{II}{A} = \overset{I}{B}$; $\overset{II}{P}\overset{II}{A} = \overset{II}{A}$, $\overset{II}{P}\overset{III}{A} = \overset{II}{B}$; $\overset{III}{P}\overset{III}{A} = \overset{III}{A}$, $\overset{III}{P}\overset{IV}{A} = \overset{III}{B}$, &c.

Jam si in æquatione pro relatione dissipationum conjugatarum modo inventa, $\frac{\overset{I}{B}\overset{I}{C}}{P\overset{I}{A}q} = \frac{2n}{\overline{N-1}\,.\,P} + \frac{BC}{PAq}$, per BC et $\overset{I}{B}\overset{I}{C}$ intelligantur successive singula paria dissipationum conjugatorum, et per PA et $P\overset{I}{A}$ distantiæ focorum conjugatorum ipsis respondentes;

 item

item per P, N et n eorum valores ſingulis lentis proprii, habebuntur tot æquationes, quot ſunt lentes: videlicet; $\overset{I}{B}\overset{I}{C} = B^2 \times \left(\frac{2n}{\overline{N-1}\,.\,P} + \frac{BC}{A^2}\right)$; $\overset{II}{B}\overset{II}{C} = \overset{I}{B}{}^2 \times \left(\frac{2\overset{I}{n}}{\overline{\overset{I}{N}-1}\,.\,\overset{I}{P}} + \frac{\overset{I}{B}\overset{I}{C}}{\overset{I}{A}{}^2}\right)$; $\overset{III}{B}\overset{III}{C} = \overset{II}{B}{}^2 \times \left(\frac{2\overset{II}{n}}{\overline{\overset{II}{N}-1}\,.\,\overset{II}{P}} + \frac{\overset{II}{B}\overset{II}{C}}{\overset{II}{A}{}^2}\right)$; $\overset{IV}{B}\overset{IV}{C} = \overset{III}{B}{}^2 \times \left(\frac{2\overset{III}{n}}{\overline{\overset{III}{N}-1}\,.\,\overset{III}{P}} + \frac{\overset{III}{B}\overset{III}{C}}{\overset{III}{A}{}^2}\right)$; et ita porro, ſi plures fuerint lentes. Ponantur puncta B, A, C, coincidere, ſive ex primo foco A prodire radium compoſitum, ut diſſipatio prima BC ſit nulla, et reducendo has æquationes, ponendo BC = o, habentur valores diſſipationum ſucceſſivarum, pro quovis lentium numero:

1. $\overset{I}{B}\overset{I}{C} = B^2 \times \frac{2n}{\overline{N-1}\,.\,P}$.

2. $\overset{II}{B}\overset{II}{C} = \overset{I}{B}{}^2 \times \frac{2\overset{I}{n}}{\overline{\overset{I}{N}-1}\,.\,\overset{I}{P}} + \frac{\overset{I}{B}{}^2\,.\,B^2}{\overset{I}{A}{}^2} \times \frac{2n}{\overline{N-1}\,.\,P}$.

3. $\overset{III}{B}\overset{III}{C} = \overset{II}{B}{}^2 \times \frac{2\overset{II}{n}}{\overline{\overset{II}{N}-1}\,.\,\overset{II}{P}} + \frac{\overset{II}{B}{}^2\,.\,\overset{I}{B}{}^2}{\overset{II}{A}{}^2} \times \frac{2\overset{I}{n}}{\overline{\overset{I}{N}-1}\,.\,\overset{I}{P}}$
$+ \frac{\overset{II}{B}{}^2\,.\,\overset{I}{B}{}^2\,.\,B^2}{\overset{II}{A}{}^2\,.\,\overset{I}{A}{}^2} \times \frac{2n}{\overline{N-1}\,.\,P}$.

4. $\overset{IV}{B}\overset{IV}{C} = \overset{III}{B}{}^2 \times \frac{2\overset{III}{n}}{\overline{\overset{III}{N}-1}\,.\,\overset{III}{P}} + \frac{\overset{III}{B}{}^2\,.\,\overset{II}{B}{}^2}{\overset{III}{A}{}^2} \times \frac{2\overset{II}{n}}{\overline{\overset{II}{N}-1}\,.\,\overset{II}{P}}$
$+ \frac{\overset{III}{B}{}^2\,.\,\overset{II}{B}{}^2\,.\,\overset{I}{B}{}^2}{\overset{III}{A}{}^2\,.\,\overset{II}{A}{}^2} \times \frac{2\overset{I}{n}}{\overline{\overset{I}{N}-1}\,.\,\overset{I}{P}} + \frac{\overset{III}{B}{}^2\,.\,\overset{II}{B}{}^2\,.\,\overset{I}{B}{}^2\,.\,B^2}{\overset{III}{A}{}^2\,.\,\overset{II}{A}{}^2\,.\,\overset{I}{A}{}^2\,.\,A^2} \times \frac{2n}{\overline{N-1}\,.\,P}$.

Et ita deinceps, ſi plures fuerint lentes.

§ 18.

§ 18.

Datis positionibus lentium quotcunque in communi axe, una cum singularum legibus refractionum pro radiis omnis generis, oporteat invenire relationem inter distantias focorum principalium earundem lentium, quæ efficiat, ut radii heterogenei à quovis puncto advenientes vel paralleli, post refractionem in omnibus lentibus emergant, sine dissipatione à diversa refrangibilitate radiorum oriunda, si fieri potest.

Solutio problematis generaliter absolvitur ponendo expressionem dissipationis, datæ multitudini lentium convenientem, § 17, nihilo æqualem, quo ipso habetur æquatio definiens relationum distantiarum focalium, P, $\overset{\text{I}}{P}$, $\overset{\text{II}}{P}$, $\overset{\text{III}}{P}$, &c. requisitam ad id, ut dissipatio in foco ultimo evanescat. Ita in casu binarum lentium habebitur: $\frac{\overset{\text{I}}{n}}{\overline{\overset{\text{I}}{N}-1}\,.\,\overset{\text{I}}{P}} + \frac{B^2}{\overset{\text{I}}{A}{}^2} \times \frac{n}{\overline{N-1}\,.\,P} = 0$; in casu trium lentium erit: $\frac{\overset{\text{II}}{n}}{\overline{\overset{\text{II}}{N}-1}\,.\,\overset{\text{II}}{P}} + \frac{\overset{\text{I}}{B}{}^2}{\overset{\text{II}}{A}{}^2} \times \frac{\overset{\text{I}}{n}}{\overline{\overset{\text{I}}{N}-1}\,.\,\overset{\text{I}}{P}} + \frac{\overset{\text{I}}{B}{}^2\,.\,B^2}{\overset{\text{II}}{A}{}^2\,.\,\overset{\text{I}}{A}{}^2} \times \frac{n}{\overline{N-1}\,.\,P} = 0$; in casu quatuor lentium: $\frac{\overset{\text{III}}{n}}{\overline{\overset{\text{III}}{N}-1} \times \overset{\text{III}}{P}} + \frac{\overset{\text{II}}{B}{}^2}{\overset{\text{III}}{A}{}^2} \times \frac{\overset{\text{II}}{n}}{\overline{\overset{\text{II}}{N}-1} \times \overset{\text{II}}{P}} + \frac{\overset{\text{II}}{B}{}^2\,.\,\overset{\text{I}}{B}{}^2}{\overset{\text{III}}{A}{}^2\,.\,\overset{\text{II}}{A}{}^2} \times \frac{\overset{\text{I}}{n}}{\overline{\overset{\text{I}}{N}-1} \times \overset{\text{I}}{P}} + \frac{\overset{\text{II}}{B}{}^2\,.\,\overset{\text{I}}{B}{}^2\,.\,B^2}{\overset{\text{III}}{A}{}^2\,.\,\overset{\text{II}}{A}{}^2\,.\,\overset{\text{I}}{A}{}^2} \times \frac{n}{\overline{N-1} \times P} = 0$; et ita porro, si plures fuerint lentes.

Ita si in æquatione pro binis lentibus, $\frac{\overset{I}{n}}{\overset{I}{N} - 1 . \overset{I}{P}} + \frac{B^2}{\overset{I}{A}{}^2} \times \frac{n}{N - 1 . P} = 0$, ponatur distantia lentium $B + \overset{I}{A} = 0$, ut evadant contiguæ, quemadmodum in vitro objectivo Dollondiano; habetur $\frac{\overset{I}{n}}{\overset{I}{N} - 1 . \overset{I}{P}} + \frac{n}{N - 1 . P} = 0$. Unde patet alterutram lentium P et $\overset{I}{P}$ faciendam esse concavam, alteram convexam, et distantias earum focales affirmative sumtas P et $\overset{I}{P}$ faciendas esse in ratione $\frac{n}{N - 1}$ ad $\frac{\overset{I}{n}}{\overset{I}{N} - 1}$, quæ ratio evadit n ad $\overset{I}{n}$, si vires refractivæ mediæ utriusque lentis fuerint æquales.

Similiter sit construendum telescopium ex tribus lentibus, quarum duæ priores P et $\overset{I}{P}$ juxta se positæ constituant vitrum objectivum, tertia $\overset{II}{P}$ sit ocularis, et potentia amplificandi exprimatur numero *m*. Quoniam lentes priores P et $\overset{I}{P}$ sunt juxta se positæ, erit earum distantia $B + \overset{I}{A} = 0$. Quoniam per constitutionem telescopii radii in primam lentem incidentes sunt paralleli, erit $B = P$, et $\frac{1}{\overset{I}{B}} = \frac{1}{P} + \frac{1}{\overset{I}{P}}$. Quoniam radii ex ultima lente emergunt paralleli, erit $\overset{II}{A} = \overset{II}{P}$. Et tandem, quia potentia amplificandi indicatur numero m, erit $\overset{I}{B} = m\overset{II}{P}$. Subſtitutis itaque his valoribus in æquatione pro tribus lentibus ſupra allata, tranſit illa in hanc: $\frac{n}{\overline{N-1}\,.\,P} + \frac{\overset{I}{n}}{\overline{\overset{I}{N}-1}\,.\,\overset{I}{P}}$ $+ \frac{\overset{II}{n}}{\overline{\overset{II}{N}-1}\,.\,m^2\overset{II}{P}} = 0$. Et hæc æquatio una cum æquatione $\frac{1}{P} + \frac{1}{\overset{I}{P}} - \frac{1}{m\overset{II}{P}} = 0$, determinat rectationem diſtantiarum focalium P, $\overset{I}{P}$, $\overset{II}{P}$, qualis ad id requiritur, ut radiorum emergentium diſſipatio à diverſa refrangibilitate nulla ſit. Comparando ſcilicet has æquationes, habetur $\left(\frac{nm}{N-1} + \frac{\overset{II}{n}}{\overset{II}{N}-1}\right)\overset{I}{P} + \left(\frac{\overset{I}{n}m}{\overset{I}{N}-1}\right.$ $\left.\frac{\overset{II}{n}}{\overset{II}{N}-1}\right) P = 0$, et $\left(\frac{n}{N-1} - \frac{\overset{I}{n}}{\overset{I}{N}-1}\right) \times m^2 \overset{II}{P}$ $+ \left(\frac{\overset{I}{n}m}{\overset{I}{N}-1} + \frac{\overset{II}{n}}{\overset{II}{N}-1}\right) \times P = 0$. Eadem ratione procedendum eſt in aliis caſibus.

S. Klingenſtierna.

Stockholm,
20 Aprilis 1760.